Laws in CSP

Joakim Ahnfelt-Rønne

This is a distillation of some of the laws from Communicating Sequential Processes by Sir Charles Antony Richard Hoare, the inventor of CSP, Hoare logic, Quicksort... and null. The book is freely available online at `usingcsp.com`.

$P \sqcap P = P$	3.2.1.L1
$P \sqcap Q = Q \sqcap P$	3.2.1.L2
$P \sqcap (Q \sqcap R) = (P \sqcap Q) \sqcap R$	3.2.1.L3
$x \to (P \sqcap Q) = (x \to P) \sqcap (x \to Q)$	3.2.1.L4
$x : B \to (P(x) \sqcap Q(x)) = (x : B \to P(x)) \sqcap (x : B \to Q(x))$	3.2.1.L5
$P \parallel (Q \sqcap R) = (P \parallel Q) \sqcap (P \parallel R)$	3.2.1.L6
$(P \parallel Q) \sqcap R = (P \parallel R) \sqcap (Q \parallel R)$	3.2.1.L7
$f(P \sqcap Q) = f(P) \sqcap f(Q)$	3.2.1.L8

$P \Box P = P$	3.3.1.L1
$P \Box Q = Q \Box P$	3.3.1.L2
$P \Box (Q \Box R) = (P \Box Q) \Box R$	3.3.1.L3
$P \Box STOP = P$	3.3.1.L4

$(x : A \to P(x)) \Box (y : B \to Q(y)) =$
$(z : (A \cup B) \to ($
`if` $z \in (A - B)$ `then`
$P(z)$
`else if` $z \in (B - A)$ `then`
$Q(z)$
`else if` $z \in (A \cap B)$ `then`
$(P(z) \sqcap Q(z))))$
	3.3.1.L5

$P \Box (Q \sqcap R) = (P \Box Q) \sqcap (P \Box R)$	3.3.1.L6
$P \sqcap (Q \Box R) = (P \sqcap Q) \Box (P \sqcap R)$	3.3.1.L7

refusals($STOP_A$) = all subsets of A (including A itself)	3.4.1.L1
refusals($c \to P$) $= \{X \mid X \subseteq (\alpha P - \{c\})\}$	3.4.1.L2
refusals($x : B \to P(x)$) $= \{X \mid X \subseteq (\alpha P - B)\}$	3.4.1.L3
refusals($P \sqcap Q$) = refusals(P) $\cup$ refusals(Q)	3.4.1.L4
refusals($P \Box Q$) = refusals(P) $\cap$ refusals(Q)	3.4.1.L5
refusals($P \parallel Q$) $= \{X \cup Y \mid X \in$ refusals(P) $\wedge Y \in$ refusals(Q)$\}$	3.4.1.L6
refusals($f(P)$) $= \{f(X) \mid X \in$ refusals(P)$\}$	3.4.1.L7
$X \in$ refusals(P) $\Rightarrow X \subseteq \alpha P$	3.4.1.L8
$\{\} \in$ refusals(P)	3.4.1.L9
$(X \cup Y) \in$ refusals(P) $\Rightarrow X \in$ refusals(P)	3.4.1.L10
$X \in$ refusals(P) $\Rightarrow (X \cup \{x\}) \in$ refusals(P) $\vee \langle x \rangle \in$ traces(P)	3.4.1.L11

$P \setminus \{\} = P$	3.5.1.L1
$(P \setminus B) \setminus C = P \setminus (B \cup C)$	3.5.1.L2
$(P \sqcap Q) \setminus C = (P \setminus C) \sqcap (Q \setminus C)$	3.5.1.L3
$STOP_A \setminus C = STOP_{A-C}$	3.5.1.L4
$(x \rightarrow P) \setminus C =$ `if` $x \notin C$ `then` $x \rightarrow (P \setminus C)$ `else if` $x \in C$ `then` $P \setminus C$	3.5.1.L5
`if` $\alpha P \cap \alpha Q \cap \alpha C = \{\}$ `then` $(P \parallel Q) \setminus C = P \setminus C \parallel Q \setminus C$	3.5.1.L6
$f(P \setminus C) = f(P) \setminus f(C)$	3.5.1.L7
`if` $B \cap C = \{\}$ `then` $(x : B \rightarrow P(x)) \setminus C = x : B \rightarrow (P(x) \setminus C)$	3.5.1.L8
`if` $B \subseteq C$ and B is finite and not empty `then` $(x : B \rightarrow P(x)) \setminus C = \sqcap_{x \in B}(P(x) \setminus C)$	3.5.1.L9
`if` $C \cap B$ is finite and not empty `then` $(x : B \rightarrow P(x)) \setminus C =$ $Q \sqcap (Q \square (x : (B - C) \rightarrow P(x)))$ `where` $Q = \sqcap_{x \in B \cap C} P(x) \setminus C$	3.5.1.L10
$\text{traces}(P_{+B}) = \text{traces}(P)$	3.5.1.L11
$P_{+B} \setminus B = P$	3.5.1.L12

$(c\,?\,v \rightarrow P) \parallel (c\,!\,v \rightarrow Q(x)) = c\,?\,v \rightarrow (P \parallel Q(v))$	4.3.1.L1
$((c\,?\,v \rightarrow P) \parallel (c\,!\,v \rightarrow Q(x))) \setminus C = (P \parallel Q(v)) \setminus C$ `where` $C = \{c.v \mid v \in \alpha c\}$	4.3.1.L2